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A TEORIA DA DETECÇÃO DE SINAIS PARA O TREINAMENTO DO FOCO ATENCIONAL: UMA PROPOSTA PARA A APRENDIZAGEM DA NOÇÃO DE LINHAS POLIGONAIS

THE THEORY OF SIGNAL DETECTION FOR TRAINING ATTENTIONAL FOCUS: A PROPOSAL FOR LEARNING THE NOTION OF POLYGONAL LINES

LA TEORÍA DE LA DETECCIÓN DE SEÑALES PARA EL ENTRENAMIENTO DEL FOCO ATENCIONAL: UNA PROPUESTA PARA EL APRENDIZAJE DE LA NOCIÓN DE LÍNEAS POLIGONALES

Joalisson Bahia Santana[1]
Gabriele Souza de Carvalho[2]
Erinaldo Ferreira do Nascimento[3]
Laerte Silva da Fonseca[4]
Maria Ambrosina da Costa[5]

Resumo

Este trabalho tem o objetivo de apresentar resultados da aplicação de uma atividade matemática para auxiliar na aprendizagem da noção de linhas poligonais através do treinamento do foco atencional baseado na Teoria da Detecção de Sinais (TDS). Os participantes desta pesquisa foram alunos do 2º ano do Ensino Médio de uma escola estadual localizada no Território de Identidade Litoral Norte e Agreste Baiano. Para a realização das atividades, os alunos foram divididos em sete grupos. Como dispositivos para a coleta dos dados foram utilizadas a observação participante e uma atividade matemática, composta pelos testes de atenção baseados na TDS e um questionário sobre a realização da atividade. Os resultados revelaram que os sujeitos refletiram sobre a importância da atenção na resolução das atividades e, de modo geral, apresentaram um bom foco atencional para analisar e distinguir os estímulos com linhas poligonais. Com a realização da atividade baseada na TDS, foi possível estimar o Tempo de Reação (TR) dos grupos e a precisão das respostas apresentadas, o que permitiu analisar os dados e estimar o *d-prime*. Constatou-se que havia dois grupos com nível médio de sensibilidade para distinguir os estímulos corretamente, dois com nível alto e três com nível baixo. Conclui-se, portanto, que o desenvolvimento de atividades baseadas em testes com a TDS pode contribuir na construção de noções geométricas, à medida que colabora no desenvolvimento atencional dos alunos.

Palavras-chave: Teoria da Detecção de Sinais. Linhas Poligonais. Foco Atencional. Atividades Matemáticas.

[1] Universidade do Estado da Bahia (UNEB). Orcid: https://orcid.org/0000-0001-6576-4085. Lattes: http://lattes.cnpq.br/1422430106866517. E-mail: joalissonbahia@hotmail.com.
[2] Secretaria de Educação do Estado da Bahia (SEDUC/BA). Orcid: https://orcid.org/0000-0002-0888-2779. Lattes: http://lattes.cnpq.br/4197647109428880. E-mail: gscarvalho22@gmail.com.
[3] Instituto Federal do Sertão Paraibano (IFSertãoPB). Orcid: https://orcid.org/0000-0003-2321-7854. Lattes: http://lattes.cnpq.br/6993470112000626. E-mail: erigremio.2013@gmail.com.
[4] Instituto Federal de Sergipe (IFS). Orcid: https://orcid.org/0000-0002-0215-0606. Lattes: http://lattes.cnpq.br/6696227477981073. E-mail: laerte.fonseca@ifs.edu.br.
[5] Universidade da Cidade de São Paulo (UNICID). Orcid: http://orcid.org/0000-0003-2570-0516. Lattes: http://lattes.cnpq.br/8544596120242697. E-mail: marianina59@yahoo.com.br

Abstract

This study aims to present the results of implementing a mathematical activity designed to support the learning of the concept of polygonal chains through the training of attentional focus, based on Signal Detection Theory (SDT). The participants were 11th-grade students (2nd year of high school) from a state school located in the "Litoral Norte e Agreste Baiano" (North Coast and Agreste of Bahia) territory. For the activities, the students were divided into seven groups. Data collection involved participant observation and a mathematical activity, which included attention tests based on SDT, followed by a questionnaire regarding the experience. The results indicate that the participants reflected on the importance of attention when completing the tasks and, in general, demonstrated a good ability to focus their attention on analyzing and distinguishing stimuli involving polygonal chains. Through the SDT-based activity, it was possible to estimate the groups' Reaction Time (RT) and the accuracy of their responses, which allowed for data analysis and the calculation of the d-prime (d') sensitivity index. It was found that two groups exhibited a medium level of sensitivity in correctly distinguishing the stimuli, while two groups showed a high level, and three showed a low level. It is concluded, therefore, that the development of activities based on SDT tests can contribute to the construction of geometric concepts by fostering the development of students' attentional focus.

Keywords: Theory of Signal Detection. Polygonal Lines. Attentional Focus. Mathematics Activities.

Resumen

Este trabajo tiene como objetivo presentar los resultados de la aplicación de una actividad matemática destinada a favorecer el aprendizaje del concepto de líneas poligonales, mediante el entrenamiento del foco atencional basado en la Teoría de la Detección de Señales (TDS). Los participantes en esta investigación fueron estudiantes de segundo año de educación secundaria (equivalente al 2º año de *Ensino Médio* en Brasil) de una escuela pública estatal ubicada en el Territorio de Identidad Litoral Norte y Agreste Baiano. Para llevar a cabo las actividades, los alumnos se dividieron en siete grupos. Como instrumentos para la recolección de datos, se emplearon la observación participante y una actividad matemática, compuesta por pruebas de atención basadas en la TDS y un cuestionario sobre el desarrollo de dicha actividad. Los resultados revelaron que los participantes reflexionaron sobre la importancia de la atención en la resolución de las tareas y, en general, mostraron un buen foco atencional para analizar y distinguir los estímulos que contenían líneas poligonales. Mediante la realización de la actividad basada en la TDS, fue posible estimar el Tiempo de Reacción (TR) de los grupos y la precisión de sus respuestas, lo que permitió analizar los datos y calcular el *d-prime*. Se observó que dos grupos presentaban un nivel medio de sensibilidad para distinguir correctamente los estímulos, dos tenían un nivel alto y tres, un nivel bajo. Se concluye, por tanto, que el desarrollo de actividades basadas en pruebas con la TDS puede contribuir a la construcción de nociones geométricas, al favorecer, al mismo tiempo, el desarrollo atencional de los estudiantes.

Palabras clave: Teoría de la Detección de Señales. Líneas poligonales. Foco atencional. Actividades matemáticas.

INTRODUCTION

The objective of this work is to present the results of implementing a mathematical activity designed to support the learning of the concept of polygonal lines through attentional focus training based on Signal Detection Theory (SDT). The learning process—which entails the acquisition of knowledge and behaviors—requires students to direct their attention toward the information that needs to be retained.

Attention can be defined as a set of cognitive processes that allow for the selection and prioritization of specific sensory stimuli, thoughts, or actions, facilitating their processing while other information is ignored or assigned lower priority. Attentional focus, in turn, is the capacity to direct and/or concentrate this set of cognitive processes on specific stimuli (Gazzaniga; Heatherton; Halpern, 2018). Attentional focus training consists of a series of systematic practices aimed at enhancing the ability to direct, sustain, and regulate attention, thereby fostering the

development of concentration on relevant stimuli or tasks, as well as the control of distractions (Tang *et al*., 2022).

Consequently, for learning to occur, it is necessary to mobilize attention through activities proposed by the teacher in the classroom. This leads to the following guiding research question: How can attentional focus be trained to support the learning of the concept of polygonal lines through a mathematical activity based on Signal Detection Theory? The study describes a specific, one-off implementation and is not designed to evaluate changes in attentional focus or learning gains.

Geometric knowledge is of extreme importance for the development and study of various mathematical concepts, and even for an individual's ability to orient themselves within the space they inhabit and to interpret the world around them. Given this significance, there has been a growing emphasis on geometry—driven by research and the development of official policy documents—that advocates for its inclusion in basic education, while also highlighting its influence across a wide range of fields and professional domains.

Furthermore, geometry contributes to the understanding of shapes, spatial relationships, and scenarios encountered in diverse areas of knowledge. Effectively teaching the subject requires appropriate teacher training, suitable materials, and strategies that allow for the discussion of the criteria students use to classify geometric figures. The role of "attentional focus" is particularly noteworthy in developing this classification ability, as it serves as a fundamental construct for the analysis of geometric figures.

Recognizing the importance of geometry in the education of students throughout their basic schooling, it is essential for educators to find ways to overcome challenges related to the teaching of geometry by exploring new methodological strategies. In this context, one can draw upon findings from cognitive neuroscience, which offer insights into how the human brain and nervous system function during the learning process—specifically regarding the phenomenon of attention, which is crucial for focusing on the information one intends to grasp.

Consequently, one might look toward theories that guide the development of attentional focus through specific testing frameworks, such as Signal Detection Theory (SDT). SDT has applications in areas involving learning, perception, and memory—most notably demonstrated in recognition tasks. According to Kellen, Nunes, and Garcia-Marques (2008), the primary objectives of Signal Detection Theory (SDT) are to estimate the strength of a signal relative to noise and to identify the response strategy employed by participants. The signal is the target stimulus of a search, while the noise consists of the variability of elements present during the search, which influences

the ability to distinguish between the presence and absence of the signal (Macmillan; Creelman, 2005).

SDT can be viewed as an approach for guiding "closed" tasks, in which a student must indicate whether or not a specific stimulus (signal) is present among a set of other elements (noise). Furthermore, using SDT-based tests allows for the assessment of participants' Reaction Time (RT)—the time interval between the presentation of a stimulus and the individual's response—which reflects the speed at which an individual can react to an event (Gazzaniga; Heatherton; Halpern, 2018).

By employing SDT-based tests that incorporate geometric concepts, one can gain insight into students' understanding of the material, their current level of learning, and the difficulties they may be facing. This is because, to perform these tests effectively—that is, to accurately discriminate target stimuli from distractors—students must possess a solid grasp of the relevant geometric concepts, or at least be in the process of actively developing them. Accordingly, this paper presents an application of SDT-based tests—focusing on the concept of simple polygonal lines—with students in a 2nd-year high school class.

GEOMETRY IN BASIC EDUCATION: A BRIEF APPROACH TO POLYGONAL LINES IN LEARNING

Geometry is a field of mathematics that emerged alongside humanity, evolving in tandem with various civilizations. It is estimated to have originated along the banks of the Nile River in ancient Egypt. It is believed that "the subject arose there out of a practical need to re-demarcate land following the annual flooding of the river valley" (Boyer, 2012, p. 29).

As humanity progressed and formal schooling emerged, this body of knowledge was integrated into educational settings to demonstrate the practical applications of mathematics to students. However, in textbooks, geometry is typically relegated to the final chapters, which creates challenges for its instruction.

In the Brazilian basic education system, geometry has historically faced difficulties—largely due to its neglect in the classroom. It is often treated as a secondary component of mathematics, relegated to the end of units or academic terms, or even overlooked and abandoned entirely (Pavanello, 1993). This neglect stems from teachers' feelings of insecurity, a lack of proper training or adequate resources, and a perceived inability to teach the subject. Furthermore, there is an over-reliance on textbooks, which present geometric concepts in a purely pragmatic and isolated manner within their final chapters (Lorenzato, 1993).

According to Chieregato and Rodrigues (2008, *as cited* in Moura; Cavalcante; Gomes, 2016), Brazil experienced a significant decline in the teaching of geometry during the 1970s, affecting both elementary and secondary education. This decline was driven by several factors, the primary one being that some teachers did not consider geometry to be important. The organization of geometry within basic education is developed in accordance with certain documents, such as the National Curriculum Parameters (NCP), which propose the incorporation of geometric content and provide general guidelines for fostering learning at each stage of schooling (Brazil, 1997). In the current societal landscape—and in light of recent curricular changes—geometry is highlighted by the National Common Curricular Base (NCCB), which standardizes the content, competencies, and skills to be developed throughout the learning process, framing geometry as a thematic unit.

It should be noted, first and foremost, that geometry in basic education aims to develop an understanding of both plane and spatial figures. Ferreira (1999) emphasizes that geometry in basic education is:

> a science that investigates the forms and dimensions of mathematical entities, or a branch of mathematics that studies shapes—both planar and spatial—along with their properties; alternatively, a branch of mathematics that examines the extent and properties of figures (plane geometry) and solids (solid geometry) (Ferreira, 1999, p. 983).

This branch of science enables an understanding of the world and one's orientation within it, ultimately transforming that space. In light of this, the NCCB highlights the field of geometry within basic education, asserting that:

> [...] Geometry involves the study of a broad set of concepts and procedures necessary for solving problems related to the physical world and various other fields of knowledge. Thus, within this thematic unit, the study of position and movement in space, as well as the shapes and relationships between the elements of planar and spatial figures, can help develop students' geometric thinking. (Brazil, 2017, p. 271).

Geometry is of great importance to the development of our society. In this sense, it aims to demonstrate the diverse applications that exist between mathematics and social life. According to Rogenski and Pedroso (2007), geometric concepts are in constant, multifaceted dialogue with everyday life—appearing in nature, the arts, architecture, and various other areas of knowledge. Consequently, the field encompasses an infinite array of content, including theorems and foundational principles, that can be learned.

Among the various geometric concepts, polygons stand out. However, to understand the emergence of polygons, it is essential to examine the processes that lead to their formation: polygonal chains. As Brousseau (1986, p. 38, our translation) notes, "it is not merely a matter of knowing definitions and theorems in order to recognize when they might be used or applied; we know full well that doing mathematics entails solving problems."

In the same vein, Mori and Onaga (2012, p. 138) state that polygonal chains can be understood as "a planar figure formed by line segments, in which no two consecutive segments lie on the same straight line." It is understood that any planar polygonal figure may consist of open or closed lines; a simple closed polygonal line, therefore, defines a polygon, which can be understood as "a planar geometric figure formed by a polygonal line" (Mori; Onaga, 2012, p. 139).

Thus, an understanding of polygonal lines contributes to the development of mathematics and other educational fields—such as the arts, where they might be used to represent an object—as well as to algebraic calculation. This leads to the concept of "planar geometric shapes with a closed boundary formed by non-intersecting line segments, known as polygons" (Souza; Pataro, 2012, p. 180).

By introducing the concept of polygonal lines, educators can provide students with opportunities to solve problems and develop their full potential. Such activities help foster specific skills, enhance cognitive perception, and promote hypothetical-deductive reasoning (Lorenzato, 1995). Consequently, polygonal lines are considered one of the most fundamental concepts to grasp in geometry. These lines form the polygons studied throughout basic education, such as triangles, squares, hexagons, and pentagons, among others.

As Gay (2014, p. 240) states, "A simple, closed, planar polygonal line divides a plane into two regions, both containing an infinite number of points, with no points in common." In the author's view, learning about these lines serves as the foundation for understanding polygons and the various ways they are represented within society. Thus, it can be argued that a closed, planar polygonal line—along with the region it encloses—forms a polygon. In this context, the discussion regarding polygonal lines and geometry extends well beyond the classroom setting. The social reality of the learners is an important factor in the learning process, as it engages their attention. The next section will address Signal Detection Theory, which examines attention as a fundamental basis for learning.

AN APPROACH TO GEOMETRY THROUGH SIGNAL DETECTION THEORY

Inadequate instruction in geometry leads to conceptual gaps for students, which in turn creates difficulties in learning various mathematical topics that rely on geometric concepts.

Neglecting the teaching of geometry can even hinder an individual's perception of the world—specifically the spatial environment in which they live—thereby impairing their ability to orient themselves within that space. This is because,

> [...] studying geometry helps students develop the ability to systematize observations and information gathered from the world around them. It also fosters reasoning skills related to the manipulation of numbers and measurements; in other words, while geometry primarily focuses on shapes, it is also essential for understanding algebra—a field often characterized by its highly pragmatic nature—and helps individuals orient themselves within their spatial environment (Souza, 2020, p. 25).

Recognizing the importance of geometry in basic education, there has been a renewed emphasis on this subject, driven both by academic research investigating the challenges inherent in its teaching and learning process, and by official educational policy documents that advocate for its inclusion by highlighting its connections to other fields of knowledge.

> Currently, we observe a greater appreciation for the teaching of geometry in Brazil, largely due to the National Curriculum Parameters (NCP). The NCP indicate that individuals in modern society must possess the capacity for geometric thinking, as everyday situations and various professions—such as engineering, architecture, mechanics, and biochemistry—require this type of knowledge (Meneses, 2007, p. 01).

To support this effort to elevate the status of geometry and meet the expectations for its mastery within the school environment—ultimately making the learning process more effective—the role of the teacher is indispensable. Thus, teachers should seek ways to make the information and knowledge conveyed in the classroom increasingly accessible to their students, encouraging them to direct their attention toward learning. To achieve this, it is essential to find methods to engage students' attention and sustain it throughout the activity at hand.

In this pursuit of strategies to capture student attention, teachers can draw upon cognitive neuroscience, which provides insights into how the human brain functions and how the phenomenon of attention is mobilized. It is important to recognize that this phenomenon is limited, as the human brain cannot simultaneously focus on all the information to which it is exposed (Cosenza; Guerra, 2011). Consequently, it is necessary to carefully select the stimuli presented to students and to train their attention through specific activities, enabling them to develop the ability to focus effectively on learning geometry.

In this context, one may look to Signal Detection Theory (SDT)—a framework used to analyze data from experiments that require the categorization of ambiguous stimuli (i.e., stimuli

with characteristics that create uncertainty, arising from either signals or noise) (Abdi, 2007, *as cited* in Rosa, 2015). This theory has gained prominence in the study of stimulus identification, as it "provides an effective method for analyzing performance in identifying signals amidst noise" (Elias, 2019, p. 28).

Signal Detection Theory (SDT) emerged within the field of electrical engineering during the Second World War. It was developed to analyze and test the capability of radar systems to detect enemy aircraft, addressing concerns regarding both human and machine error, with the goal of simultaneously maximizing correct detections and minimizing false alarms (Kellen; Nunes; Garcia-Marques, 2008).

In 1996, SDT was adapted for psychophysics—applied to human sensation and perception—to help elucidate individual behavior during decision-making tasks. Subsequently, it began to be used in areas involving learning, human retention, and memory. Later, it was also applied within cognitive psychology to explain how individuals discriminate signals from noise, enabling this process to be quantified through specific measures (Kellen; Nunes; Garcia-Marques, 2008). To this end, SDT aims to estimate *d-prime* (*d'*), a value that indicates the strength of a given signal (Rosa, 2015). *D-prime* is an SDT statistical index that measures an individual's perceptual sensitivity—that is, their ability to distinguish a relevant signal from noise (Ebert *et al.*, 2024).

> Signal Detection Theory (SDT) encompasses four possible outcomes based on the presence or absence of a stimulus and whether or not that stimulus is detected. It characterizes an individual's attempt to detect a signal—a target stimulus [...]. First, "hits" (also known as "true positives") occur when the presence of a target is correctly identified. Second, "false alarms" (also known as "false positives") occur when the presence of a target is incorrectly identified, even though the target is, in reality, absent. Third, in the case of misses (also called "false negatives"), an error occurs when the presence of a target fails to be identified. Fourth, in the case of correct rejections (also called "true negatives"), the absence of a target is correctly identified (Sternberg, 2008, p. 123).

The four possible outcomes, presented in Table 1, make it possible to determine whether an individual is capable of discriminating between the stimuli to which they are exposed and providing the appropriate response to a target stimulus, with the intent of achieving a correct detection or a correct rejection. When an individual fails to achieve one of these two positive outcomes, they encounter an error in the detection of the target stimulus—which, in the context of learning, may represent a conceptual gap.

Table 1 – Reference matrix for possible SDT outcomes.

Target Stimulus (signal)	**Detected**	**Not Detected**
Presence	Hit	Miss
Absence	False alarm	Correct rejection

Source: Adapted from Sternberg (2008).

Based on the four possible outcomes in a reference matrix—using the descriptors "Hit" and "False Alarm"—one can analyze the sensitivity level of participants in a test using the SDT by calculating the *d-prime* (*d'*) index. Representing "Hit" and "False Alarm" as A and FA, respectively, *d'* can be defined as the difference between the hit rate and the false alarm rate: d' = Z(A) – Z(FA), where Z is the standardized score (z-score) from the standard normal distribution (Kellen; Nunes; Garcia-Marques, 2008).

For small sample sizes, the log-linear correction proposed by Hautus (1995) is often applied. In this approach, the corrected hit rate is calculated as Ac = (A + 0.5)/(Ns + 1), and the corrected false alarm rate as FAc = (FA + 0.5)/(Na + 1), where Ns is the number of stimuli containing the signal, and Na is the number of stimuli that do not contain the signal. These corrected rates are then used in the *d-prime* equation: *d'* = Z(Ac) – Z(FAc).

Regarding the interpretation of the d' value: *d'* = 0 indicates an inability to discriminate between signal and noise; *d'* > 0 indicates an ability to discriminate between signal and noise, with the capacity for discrimination increasing as the index value rises; and *d'* < 0 suggests atypical performance or difficulty with the task (or with data recording), requiring caution in its interpretation (Macmillan; Creelman, 2005).

In her research, Rosa (2015) estimated participants' discriminative capacity—based on the d-prime index and the strength of signal detection—by categorizing them into high, medium, and low-performance groups. High performance occurs when individuals successfully recognize stimuli through correct hits and correct rejections; medium performance is observed when subjects experience some difficulty in identification, resulting in false alarms; low performance is characterized by significant inaccuracy, where individuals fail to identify stimuli while simultaneously producing false alarms.

Furthermore, it is possible to assess the quality of a subject's response—a metric known in cognitive neuroscience as Reaction Time (RT), which refers to the speed at which a subject responds to an event (Gazzaniga; Heatherton; Halpern, 2018). Analyzing RT allows for an estimation of processing speed and provides insight into the level of difficulty encountered when

distinguishing between stimuli. For instance, an individual who completes a discrimination task with a high rate of correct hits and rejections—while maintaining a short RT—likely possesses a strong capacity for stimulus discrimination, indicating more efficient processing for that specific task. In the context of learning, accurate responses coupled with a fast RT may suggest that the subject is confident in their grasp of the relevant concepts and is focused on applying them to the activity, reflecting efficient cognitive processing.

Conversely, a subject who requires a longer RT to achieve correct hits and rejections may be experiencing uncertainty regarding the concepts, a lack of attentional focus on the task, or perhaps an excess of caution in their decision-making. Meanwhile, an individual whose RT varies significantly across trials may be exhibiting inconsistent attentional performance, frequent distractions, or difficulties in processing—all of which may point to a high level of conceptual difficulty.

Applying these insights—derived from test results using the SDT —to the learning of geometry, one could design activities that direct students to search for specific target stimuli within various figures, determining whether or not a given target is present. The responses provided by students can foster reflection on their grasp of specific concepts, while also revealing whether they are experiencing any particular level of difficulty. Furthermore, such activities can serve to train attentional focus and reinforce the geometric notions that the curriculum aims to develop.

The SDT was not originally developed for an educational context; indeed, it has been applied in recent research on decision-making regarding entrepreneurial opportunities (Mendes, 2022; Santana, 2019), as well as in studies on human rationality in decision-making (Pinker, 2022). Both areas explore the four possible outcomes of the decision-making process, prompting reflection on which choices are appropriate. This suggests that the SDT is highly applicable to human decision-making—and, by extension, to the educational context—given that students are constantly required to make decisions while completing school-based tasks.

In this regard, it is essential for educators to seek out alternative theories and dynamics for use in the classroom—especially those grounded in the principles of cognitive neuroscience, which account for how the human brain functions. After all, cognitive neuroscience is increasingly gaining traction within the field of teaching and learning; by examining these processes through the lens of brain function, it provides a scientific basis for various learning theories, thereby earning its place within the school community (Fonseca, 2015).

METHODOLOGY

With the aim of stimulating attentional focus to support the learning of the concept of polygonal lines—through a mathematical task based on Signal Detection Theory (SDT)—a quantitative field study was conducted with a class of 11th-grade (2nd year of high school) students at a public school in the state of Bahia.

This research employs a quantitative approach, which, according to Pereira and Ortigão (2016), seeks to investigate the intensity and distribution of specific characteristics within a population, while also contributing to the understanding of social issues. Although such an approach can yield results with varying degrees of generalizability, the analysis becomes more robust when the contexts in which the data were generated are taken into account. Thus, it is possible to examine the observable, concrete data provided by the research participants to determine Reaction Time (RT) and the *d-prime* (*d'*) metric, thereby allowing for inferences regarding the students' ability to distinguish between stimuli.

Regarding data collection procedures, this study is classified as field research, as the data were generated directly within the context under investigation. "Field research in education, therefore, is characterized by the researcher entering the field—the educational spaces—to collect data, with the objective of understanding the phenomena that occur therein" (Tozoni-Reis, 2009, p. 28). This is the approach taken here, as the study involves entering the educational space of the classroom to examine student performance on mathematical tasks based on SDT.

The selection of this specific class is due to the fact that one of the authors of this article teaches the group, which meant that the implementation did not require the consent of other instructors. Furthermore, this choice is also based on the expectation that high school students should already possess prior knowledge of polygonal lines, as this topic is covered during the final years of elementary/middle school (Ensino Fundamental). The task used in this research emerged from the studies and discussions of the Research Group on the Neurocognitive Development of Mathematical Learning (neuroMATH[6]). One of the group's objectives was to develop a task incorporating a theory of selective attention; this led to the idea of addressing polygonal lines through the lens of Signal Detection Theory.

[6] Research group at the Instituto Federal de Sergipe that holds monthly meetings to reflect and discuss theories of attention in favor of learning science and mathematics. Regarding the theory in question, the meeting took place on 05/04/2022 with the presentation of its theoretical principles and also proposals for activities in the form of duels to gain knowledge and improve the concepts of the four theories of attention.

To guide the implementation of the task with the class participating in the study, the process was structured into three stages: introduction of the task (1), completion of the activity (2), and discussion of the results (3). This follows the framework of a mathematical inquiry lesson, in which the teacher shifts from the role of a "transmitter" of knowledge to that of a mediator—sparking students' curiosity and encouraging them to seek answers to the problem. "The teacher remains the key element [...] in these lessons, with the responsibility of helping students understand what it means to conduct an inquiry and how to go about it" (Ponte; Brocardo; Oliveira, 2003, p. 1).

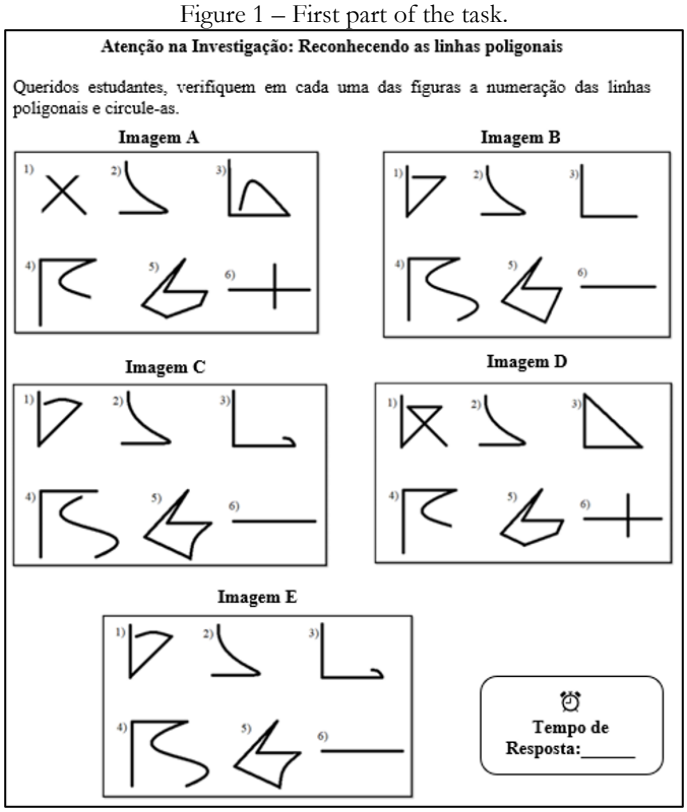
As this study stems from a purely pedagogical, instructional activity conducted within a regular classroom setting—without any intervention or alteration to the students' routine—it meets the criteria for exemption from submission to and review by the CEP/CONEP system (Research Ethics Committee/National Research Ethics Commission), as stipulated in Article 1, Item VII, of Resolution No. 510/2016 of the National Health Council (Brazil, 2016). It should be noted that the analysis of written work and group discussions was conducted on a strictly anonymous and aggregated basis, making it impossible to identify any participating student—either directly or indirectly—thereby ensuring the absolute confidentiality of the information.

Data collection involved the use of students' written work (via a printed task sheet) and observations made by the teacher facilitating the activity. Before beginning, the students were informed about the purpose of the activity and that their responses would be used as research data; all participants consented to take part. Following this, the task was administered according to the procedures described below.

Twenty-eight students participated in the study, organized into seven groups of four members each. Seven collective responses were analyzed, with each group serving as the unit of analysis. Therefore, the results represent the performance of the groups and cannot be attributed to individual students.

In the first stage, an introduction was provided by explaining the concept of polygonal lines; the lesson began with a review of the concept of polygons, leading into the structure of polygonal lines. Following this, the class was invited to carry out the task, with students divided into 7 groups of 4 members each. The next stage involved working on the assigned activity, starting with the first part of the task (Figure 1).

Figure 1 – First part of the task.



Source: The authors (2022).

In this first part, a 5-minute time limit was set for students to conduct a visual search for specific elements within the figures. It should be noted that, even after the 5 minutes had elapsed, students were permitted to continue working and signal once they had finished, allowing for the recording of the total time taken. The 5-minute duration was chosen because it was deemed sufficient for group members to analyze each of the stimuli present in Images A, B, C, D, and E, engage in brief discussions, and finalize their answers—effectively allowing an average of 1 minute per image.

This initial phase of the activity was crucial for the teacher to verify whether the students had understood the instructions. "Interpreting the task should, in itself, be one of the objectives of these lessons; therefore, it is expected that, over time, students will learn to carry out this process autonomously or in collaboration with their peers" (Ponte; Brocardo; Oliveira, 2003, p. 3).

An examination of the images used in the tests reveals that Images A, B, C, and D contain simple polygonal lines—that is, they include the target stimulus—whereas Image E does not contain any simple polygonal lines and, consequently, lacks the target stimulus. Thus, the ideal outcome for these tests involves correctly identifying the stimulus in four of the images and

correctly rejecting it in the fifth. Once the time allotted for the first part had expired, the students proceeded to Part 2 of the task (Figure 2).

Figure 2 – Second part of the task.

Registro das respostas e correção

Imagens	Linhas poligonais identificadas	Acertos	Erros
A			
B			
C			
D			
E			

Quais estratégias foram utilizadas para identificar as linhas poligonais?

Vocês tiveram mais:
( ) Acertos ( ) Erros

Se houve erros, vocês acreditam que foram por quais motivos?

O que vocês acharam da atividade?

Source: The authors (2022).

This second part of the task was designed to enable an in-depth analysis of the students' responses, allowing for an understanding of the reasoning employed in solving the problems, as well as the collection of their perceptions regarding the task. Consequently, this phase serves as an opportunity for students to reflect on what was accomplished in Part 1; for this reason, no time limit was imposed for completing it.

In the final stage, the results were discussed with the groups. This discussion involved analyzing the responses each group had entered into the table—comparing them against the concept of polygonal lines—with the students themselves examining their own entries to identify correct and incorrect answers. The next section provides a detailed analysis of the task, grounded in the Theory of Didactic Situations (TDS) and utilizing a response-pairing technique.

For the calculation of *d-prime* (*d'*), the study accounted for "hits" and "false alarms" regarding the presence or absence of the target stimulus in each image. In other words, these metrics were used to assess the complete accuracy of the students' responses for each image in the task. It should be noted, however, that failing to achieve a "complete hit" does not necessarily imply a total error; rather, it may reflect a "partial hit." Such partial hits—observed during the analysis of individual images—may indicate conceptual gaps or a lower level of attentional focus during the

activity, which can be interpreted as a partial development of the learning related to the concepts being addressed.

Furthermore, given the small number of images (5) used in the search task, the Hautus (1995) log-linear correction was applied. Specifically, Ns = 4 (representing the four images containing the target stimulus) and Na = 1 (representing the single image that did not contain the target stimulus).

The use of a limited number of stimuli is justified by the short duration typically available for classroom activities that require the engagement of attention. However, this activity can serve as a starting point for other similar practices, allowing for greater variability in the search elements and increased uniformity in the images that lack stimuli. In this article, the use of images containing stimuli was prioritized, as it was deemed more relevant—at this stage—for students to be able to identify which elements were, in fact, polygonal lines.

Regarding the determination and analysis of reaction time (RT), it is worth noting that the perspective adopted here focuses on the time taken by the groups to identify the stimuli, as well as the time spent discussing and providing answers. This differs from the conventional definition of RT, as it also accounts for the time spent on group discussion.

ANALYSIS OF THE ACTIVITY'S DEVELOPMENT

Based on the concept that polygonal lines are a set of successive, non-collinear line segments, students were asked to identify the polygonal lines present in the five images in Part 1 of the task (Figure 1). The seven groups investigated the polygonal lines, but the process took more than 5 minutes, as shown in Table 2 below.

Table 2 – Reaction Time (RT) for Part 1 of the task.

Group	Reaction Time
1	7’27”
2	8’47”
3	9’40”
4	9’26”
5	8’32
6	9’16”
7	7’56”

Source: Research data (2022).

It is observed that the shortest response time (RT) was 2 minutes and 27 seconds longer than the stipulated minimum, while the maximum RT was 9 minutes and 40 seconds—a difference of 2 minutes and 13 seconds between the group with the longest RT and the group with the shortest

RT. This time tracking was implemented to assist in the analysis of the students' RT; in the context of the SDT, this duration can be used to draw inferences regarding the subject's mastery of a concept and/or their level of attentional focus, as well as the speed at which they respond to a stimulus. Table 3, presented below, shows the responses recorded by each group during the search task.

Table 3 – Responses from the participating groups.

Groups	Images				
	A	**B**	**C**	**D**	**E**
1	1, 5, 6	1, 3, 5	4	1, 3, 5	1, 6
2	1, 5, 6	1, 3	1, 5	1, 3, 5	None
3	5	1, 3, 5	None	1, 3, 5	None
4	1, 5, 6	1, 3, 5	3, 5	1, 3, 5, 6	None
5	1, 5	3, 5	4	1, 3, 5	None
6	1, 5, 6	3, 5, 6	1, 5, 6	1, 3, 5	1, 2, 5
7	1, 5, 6	1, 3, 5, 6	1, 5	1, 3, 5, 6	None

Source: Research data (2022).

By comparing Tables 1 and 3, it can be observed that for Image A, only Group 3 correctly identified the polygonal lines without any errors. The other groups failed by incorrectly identifying lines 1 and/or 6 as polygonal lines.

For Image B, Groups 1, 3, and 4 correctly identified lines 1, 3, and 5 as polygonal lines without any errors. Group 2 failed to detect line 5; Group 5 failed to detect line 1; Group 6 also failed to detect line 1 and additionally failed to detect line 6; and Group 7, while making correct identifications, also failed to detect line 6.

Groups 1 and 5 correctly identified the polygonal lines in Image C, whereas the other groups failed, as they included lines that are not polygonal. In Image D, all groups correctly identified lines 1, 3, and 5 as polygonal lines; however, Groups 4 and 7 failed to detect line 6. Image E is the only one in which the target stimulus is absent; consequently, Groups 1 and 6 produced false alarms, while the remaining groups correctly rejected the presence of any polygonal lines.

Although Group 1 achieved the shortest resolution time, it failed to detect the target stimulus in Image A and produced a false alarm for Image E. Group 2 experienced failures with Images A and D, and failed to detect a signal in Image B; Group 3 missed a signal in Image D; and Group 4 failed on Images A and B, while also missing a signal in Image D.

Group 5 failed on Image A and missed a signal present in Image B. Group 6 failed on Images A and B, experienced both a failure and a missed signal in Images B and D, and produced

a false alarm for Image E. Finally, Group 7 failed on Images A, B, and C, and experienced both a failure and a missed signal for Image D.

Based on these results, Group 3 came closest to achieving a perfect score, failing only to detect signals in Image D. However, this group also took the longest time to complete the search; this suggests that the students were careful in their choices—which led to a higher number of correct identifications—at the cost of requiring more time.

It is also possible to analyze the participants' ability to distinguish between the stimuli—consisting of polygonal lines—in the tests performed, by examining the percentages for the four possible outcomes of the SDT, as shown in Table 4 below.

Table 4 – Percentage of student responses according to possible SDT outcomes.

	Images A, B, C and D.				**Image E.**			
Group	**Correct responses / (%)**		**Incorrect responses/ (%)**		**False alarms / (%)**		**Correct rejections / (%)**	
1	3	75%	1	25%	1	100%	0	0%
2	1	25%	3	75%	0	0%	1	100%
3	3	75%	1	25%	0	0%	1	100%
4	1	25%	3	75%	0	0%	1	100%
5	2	50%	2	50%	0	0%	1	100%
6	1	25%	3	75%	1	100%	0	0%
7	0	0%	4	100%	0	0%	1	100%

Source: The authors (2022).

Taking into account the students' accuracy rates, it is possible to conduct analyses based on their sensitivity—that is, their ability to distinguish between stimuli—and, consequently, to determine the *d-prime* values for the groups that participated in the tests, as well as to establish their level of sensitivity, as shown in Table 5.

Table 5 – Analysis of the ability to distinguish between stimuli.

Group	**Ac**	**AFc**	***D-prime***	**Sensitivity level**	**Overall percentage of correct responses**
1	0,70	0,75	−0,15	Low	60%
2	0,30	0,25	0,15	Medium	40%
3	0,70	0,25	1,20	High	80%
4	0,30	0,25	0,15	Medium	40%
5	0,50	0,25	0,67	High	60%
6	0,30	0,75	−1,20	Low	20%
7	0,10	0,25	−0,61	Low	20%

Source: The authors (2022).

Based on Table 5, it can be observed that: groups 1, 6, and 7 showed negative values, indicating poor performance in discriminating between images with and without the target signal; groups 2 and 4 showed values close to zero, suggesting an average discrimination capacity; and groups 3 and 5 showed the highest positive values, indicating the best relative discrimination among the groups evaluated.

Thus, it is evident that groups 1, 6, and 7 exhibited a low level of sensitivity, which reflects inaccuracy in their responses—or, more specifically, significant difficulty in identifying polygonal lines. Groups 2 and 4 showed an average level of sensitivity, indicating some difficulty in identifying the polygonal lines. In contrast, groups 3 and 5 demonstrated a high level of sensitivity, meaning they were accurate in identifying the polygonal lines. Overall, this suggests that most of the participating students have developed the ability to identify polygonal lines, as well as the capacity to focus their attention on the task at hand.

However, it is important to interpret these findings with caution, as each group evaluated only one image that did not contain the target signal. Consequently, the results are highly sensitive to the occurrence of even a single false alarm, which precludes drawing robust conclusions regarding the participants' attentional capacity.

In addition to the *d-prime* analysis, the overall percentage of correct responses in the image analysis can be examined: 20% for groups 6 and 7, 40% for groups 2 and 4, 60% for groups 1 and 5, and 80% for group 3. It should be noted that the percentage of correct responses was based on the participants' ability to correctly answer the tests involving five images, with each question accounting for 20% of the total. It is worth noting the partial correct responses in the analysis of the elements that make up each image; although these are not factored into the *d-prime* calculation, they indicate a certain level of sensitivity in discriminating between stimuli—that is, some degree of conceptual understanding and application of attentional focus.

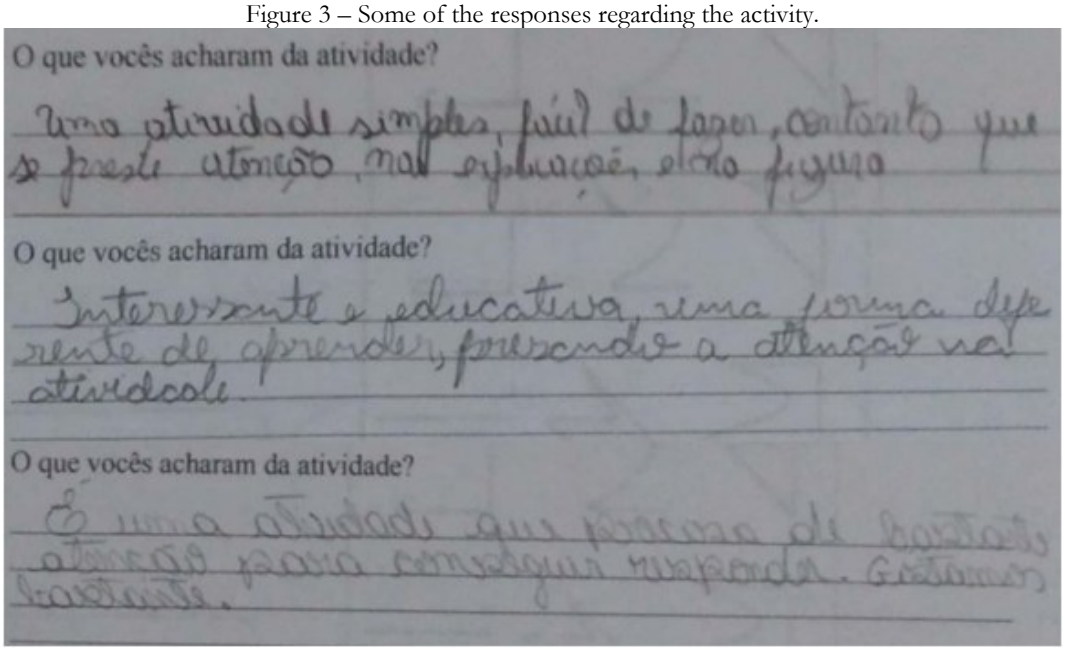
Given this, and considering that the concept of polygonal lines was introduced only shortly before the visual search task was presented, the task can be viewed as engaging and well-suited for developing students' attentional focus, even though no group achieved a perfect score.

The task also included a question regarding the reasons why the students believed they had made errors in identifying the polygonal lines. Groups 1, 2, 3, 4, and 7 cited a lack of attention as one of the reasons for their errors, further noting that some examples encountered were not covered in the teacher's explanation. This suggests a potential reliance on the teacher's specific examples, combined with a lack of careful attention to the underlying concept of polygonal lines.

Groups 5 and 6 indicated that their errors were related to how they perceived the image; Group 5 explained, "*It was because they touched each other, but unfortunately, they didn't continue*" while Group 6 stated, "*Because I couldn't correctly identify some of the polygons*".

These accounts show that the students had not yet fully grasped the concept of polygonal lines and remained fixated on the specific examples they had been shown. Another important factor to consider is that this task differed from the type of work the students were accustomed to—it was their first encounter with this kind of activity, which also required the use of selective attention. Finally, the students were asked for their thoughts on the activity. Figure 3, below, presents the responses from some of the groups that participated in the study.

Figure 3 – Some of the responses regarding the activity.



Source: Research data (2022).

Based on the responses provided, it can be inferred that the students enjoyed the activity and recognized the need to exercise attention and understand the concept of polygonal lines in order to carry out the visual search. Also noteworthy is the reflective nature of the evaluation questions, which encourage students to think about their own strategies and their impressions of the activity. Reflecting on their actions while completing the tasks can help students identify the paths they should take, the adjustments required, and the areas in which they need to improve to consolidate their knowledge—a process that differs from simply receiving direct guidance from their teachers.

FINAL CONSIDERATIONS

This article presents the results of a mathematical activity designed to support the learning of the concept of polygonal chains by training attentional focus, based on Signal Detection Theory (SDT). The task was administered to a group of 11th-grade (2nd year of high school) students at a public school in the state of Bahia.

By calculating the *d-prime* (*d'*)values for the participants, it was determined that three groups showed a low level of sensitivity in distinguishing stimuli containing polygonal chains, two groups showed a medium level, and two groups reached a high level. These data indicate that the majority of the participating students have demonstrated good development of attentional focus and a solid grasp of the concept of polygonal chains; generally, they were able to accurately identify the presence of polygonal chains and correctly reject their absence. Thus, it can be inferred that the students are successfully learning the concept of polygonal chains, and that their attentional focus is playing a key role in their performance on these tasks.

It should also be noted that the participants' response times (RT) remained high, as all groups exceeded the 5-minute limit. However, it is important to consider that the concept of polygonal chains was previously unfamiliar to the students, which necessitated more time for analysis—a challenge that could be addressed through a series of progressive activities focused on this concept.

Regarding the students' own assessment of the activity, it is evident that they recognized the importance of engaging and focusing their attention to achieve success. In other words, they realized that both attentional focus and a mastery of the underlying concept are fundamental to performing well on such tasks. Thus, the importance of practices such as the one developed in this research can be highlighted; it offers an alternative activity based on tests using the SDT to facilitate the construction of geometric concepts—and the verification of such constructions—by engaging the students' attentional focus. Furthermore, it encourages reflection on the role of attention, helping students realize and understand how this mechanism is crucial for carrying out their tasks and, consequently, for achieving effective learning.

For future research, building upon the use of attention tests based on SDT, one could envision the implementation of more sophisticated tests—incorporating a larger set of images and administered on an individual basis. This would allow for the calculation of each participant's d-prime value and reaction time (RT), enabling increasingly precise analyses of students' attentional focus while they engage in mathematical problem-solving activities.

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